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On the Spherical Symmetry of a Static Perfect Fluid*

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Abstract. A globally static space-time with asymptotically Euclidean behavior representing a finite body of a perfect fluid and a vacuum region is shown to be diffeomorphic to Euclidean space and its metric spherically symmetric whenever the magnitude of the gravitational field strength is only a function of the gravitational potential. Under some additional physical assumptions it is then proved that this spherically symmetric solution is not deformable, that is, does not admit a nontrivial first order perturbation that is also a static, asymptotically Euclidean perfect fluid with the same equation of state and the same central value of the pressure and the gravitational potential.

1. Introduction

This paper deals with the equilibrium configurations of a nonrotating perfect fluid in a general relativistic framework. As in the Newtonian theory, where it was proved a long time ago (see Carleman [5] and for the more general context Lichtenstein [14]), this equilibrium was always believed to be attained for a spherically symmetrical configuration. The proof in the Newtonian theory used potential theory on three dimensional Euclidean space in a way that probably cannot be generalized to Riemannian manifolds.

Avez [1 and 2] suggested a very elegant argument based on Morse theory which, however, is only applicable in the very restricted case, where the gravitational potential U has only nondegenerate critical points and the magnitude W of the gravitational field strength, i.e. the length of the three-dimensional gradient of U, is a function of U only. Under these assumptions it follows from the asymptotically Euclidean behavior that the 3-space is in fact diffeomorphic to $\mathbb{R}^3$ and the space-time is spherically symmetric, i.e. admits $SO(3)$ as a isometry group with two-dimensional spacelike orbits.

The main result of this paper confirms the conjecture that the latter conclusion is probably independent of these restrictive hypotheses.

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