Commun. math. Phys. 20, 57—84 (1971)

A Class of Homogeneous Cosmological Models III: Asymptotic Behaviour

M. A. H. MacCallum

Department of Applied Mathematics and Theoretical Physics, University of Cambridge

Received August 14, 1970

Abstract. The behaviour near to and far from an initial singularity in a broad subclass of the models studied in previous papers [1–3] is examined. The influence of the matter on the evolution at these times is discussed. The singularity types for the various models, which are mostly of cigar or oscillatory nature, are found. It is discovered that among these models, only those of the same Bianchi type as a Robertson-Walker model can become "approximately Robertson-Walker" in a sense defined in the paper. Qualitative conclusions concerning black-body isotropy, the Hubble relation, helium abundance and horizon structure are given.

1. Introduction

This paper is concerned with the asymptotic behaviour of a class of homogeneous cosmological models studied in previous papers [1–3]. These are spacetimes which satisfy Einstein's field equations for a perfect fluid and admit a three-parameter group of motions G_3 simply-transitive on spacelike sections orthogonal to the fluid flow vector u^a. The only spacetimes with three-surfaces of homogeneity orthogonal to u^a which do not admit a simply-transitive three-parameter group of motions are those of Case I of Kantowski and Sachs [4].

All non-stationary spatially homogeneous cosmological models (including those in which the matter rotates or accelerates) have singularities, provided that the cosmological constant Λ does not have a large positive value [5]. (These singularities are unambiguously physical singularities if the energy density of matter μ is everywhere positive, and its pressure p is not negative, as will be assumed throughout this paper.) For the models considered here one may deduce the existence of the singularity from the (0 0) field equation, Raychaudhuri's equation. Defining

$$u_{a;b} =: \theta_{ab} =: \tfrac{1}{3}\theta h_{ab} + \sigma_{ab}$$