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Classes of Operations in Quantum Theory

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Abstract. Recent work of Davies and Lewis has shown how partially ordered vector spaces provide a setting in which the operational approach to statistical physical systems may be studied. In this paper, certain physically relevant classes of operations are identified in the abstract framework, some of their properties are derived and applications to the Von Neumann algebra model for quantum theory are discussed.

§ 1. Introduction

In a previous paper [7], the operational approach to the theory of statistical physical systems, originally suggested by Haag and Kastler [13] and recently formulated in terms of partially ordered vector spaces by Davies and Lewis [4], was discussed in some detail.

Briefly, the abstract formulation may be described as follows. Regarding states as (equivalence classes of) statistical ensembles of the physical system under examination, the set of states may be represented by a generating cone K for a real vector space V, where addition of elements of K represents a process in which the corresponding states are mixed and where multiplication by positive scalars represents the proportional increase in the number of copies of the system in the corresponding state. The element 0 of K represents the state in which there are no systems. A state f may be thought of as a beam of systems emerging from some conditioning apparatus at a constant rate $e(f)$. Then, e extends to a strictly positive linear functional, called the strength functional, on V and the set B of states f such that $e(f)=1$ forms a base for the cone K. B is said to be the set of normalized states of the system. The Minkowski functional on the convex hull of $B \cup (-B)$ defines a semi-norm on V which coincides with e on K. The assumption that countable mixtures of states may be formed leads to the conclusion that this semi-norm is, in fact, a norm with respect to which V is complete [8]. In general, K need not be closed for the norm topology although a result of Ellis [11] shows that if $\bar{K}, \bar{B}$ are the completions of K, B respectively, then $\bar{K}$ is a cone in V with base $\bar{B}$ and that the associated