

Collisionless Radiation in Closed Cosmologies

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Received August 14, 1970

Abstract. An outline is given of some new results for collisionless radiation in Bianchi type IX cosmologies. An interesting new effect is displayed which distorts the pattern of any microwave temperature anisotropy from the quadrupole + dipole shape previously predicted in these models.

I. Introduction

This paper presents an outline of several new results for rotating homogenous anisotropic cosmologies containing collisionless radiation. A fuller account has been submitted for publication elsewhere.

We concentrate on Bianchi type IX homogeneous cosmologies [1–5] which contain collisionless radiation (“neutrinos”) and in which there may be rotation. A type IX metric may be written [3]

$$ds^2 = -dt^2 + R^2(t) e_{ij}^{2\beta(t)} \sigma^i \sigma^j, \quad (1.1)$$

where β_{ij} is a 3×3 symmetric traceless matrix, not in general diagonal, and the σ^i obey the curl relations of type IX [2, 3]:

$$d\sigma^i = \frac{1}{2} \varepsilon_{ijk} \sigma^j \wedge \sigma^k. \quad (1.2)$$

In dealing with collisionless radiation we write the homogenous stress tensor [6–8]

$$T_{\mu\nu} = R^{-3} \int F(q_i, t) q_\mu q_\nu (q^0)^{-1} d^3q \quad (1.3)$$

where the components q_α are expressed in the $\{dt, \sigma^i\}$ basis of Eq. (1.1) and $(q^0)^2 = R^{-2} e_{ij}^{-2\beta} q_i q_j$. Also, $d^3q = dq_1 dq_2 dq_3$.

For a collisionless system $F(q_i, t)$ is a solution of the collisionless Boltzmann equation, and hence is a function only of constants of the geodesic motion [7]. F therefore can be expressed solely as a function of the three quantities k_i :

$$k_i \equiv q_i(t_0) \quad (1.4)$$