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Neutrino Radiation in Gravitational Fields

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Abstract. A differential equation representing radiation solutions of the general relativistic Weyl equation is derived. Their optical properties and the group of motion of the corresponding energy-momentum tensor are studied. If there exists neutrino radiation the Riemann space must be algebraically special and the propagation of the neutrinos occurs only along one of the principal null directions. Gravitational- and neutrino *pp*-waves taken together, represent an exact solution of the Weyl-Einstein system of field equations.

§ 1. Introduction

The physics of neutrinos is determined by two interactions: the weak interaction dominating within elementary particle physics, and the long-range gravitational interaction. The latter is most important for some stages of stellar evolution (e.g. [1]), at the beginning of our Universe (e.g. [2]), and in connection with a cosmic neutrino background radiation (e.g. [3]).

In the framework of General Relativity, neutrinos are commonly treated phenomenologically as point particles moving with the speed of light and show in this simplified model, the same properties as photons. On the other hand, a rigorous quantum-mechanical treatment in curved spacetime must start from the generally covariant Weyl theory of the neutrino (§ 2). In the following, the characteristic properties of a special case of Weyl's theory in a 4-dimensional Riemann space shall be studied with *rigorous* methods in analogy to the electromagnetic case. Several deviations from the phenomenological description are found in this way.

We restrict ourselves to *neutrino radiation fields* defined in analogy to electrodynamics and we derive the differential equation for such solutions of Weyl's equation in § 3. By means of this equation, we determine the optical properties of the null-congruence given by the 4-vector j^α of the probability-density and the group of motion of the energy-momentum tensor. After this, it is possible to classify the neutrino radiation solutions by means of some supplementary conditions in § 4. We show in § 5 the algebraic conditions of the metric field, which must be satisfied in order that the corresponding solution can exist. Especially, there