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Event Horizons in Static Scalar-Vacuum Space-Times

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Abstract. The following theorem is established. Every zero-mass scalar field which is gravitationally coupled, static and asymptotically flat, becomes singular at a simply-connected event horizon. In the special case where the gravitational coupling of the scalar energy density is neglected, the solutions are computed explicitly. Some properties of the singular event horizons are discussed, and a brief mention of related work with non-static scalar fields is given.

1. Introduction

Recent interest in the theory of gravitational collapse has raised many questions regarding the existence and nature of event horizons in relativity. Some definite results are known. Israel has shown [1] that for the class of asymptotically flat, static vacuum fields, only the spherically symmetric Schwarzschild solutions with $m \geqq 0$ have a regular event horizon ($r = 2m$), and [2] that for the corresponding electrovac space-times, the Reissner-Nordström solutions with $m \geqq G^{\frac{1}{2}}|e|/c$ are the only ones with non-singular horizons. In view of these special cases, it is therefore natural to ask whether, for arbitrary, asymptotically flat static fields, a regular event horizon is destroyed by any asymmetric perturbation due to sources within the surface $g_{00} = 0$.

In this connection there has been some recent interest in another special class – namely, the static coupled gravitational and massless scalar fields (where by "massless scalar field" we mean a scalar field for zero-mass particles). The spherically symmetric solution of Janis, Newman, and Winicour (JNW) [3] has the interesting property that the event horizon is a singular point in the space no matter how small the coupling constant becomes. Penney [4] has suggested that this surprising result is due to the imposition of spherical symmetry, and that, by considering asymmetric solutions, one is led to a nonsingular horizon. However, his example in support of this contention contains an error [5], and, in fact, his solution is singular at the horizon.

In this paper we propose to clear up much of the controversy about event horizons associated with asymptotically flat, static, massless scalar