Commun. math. Phys. 19, 204—218 (1970)
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Statistical Mechanics of Quantum Mechanical Particles with Hard Cores

II. The Equilibrium States

SALVADOR MIRACLE-SOLE
C.N.R.S., Marseille

DEREK W. ROBINSON
Centre Universitaire, Luminy, Marseille

Received July 12, 1970

Abstract. The states of a quantum mechanical system of hard core particles are characterized as a convex weak * compact subset of the states over a C^* algebra associated with the canonical (anti-) commutation relations. It is shown that the mean conditional entropy, i.e. entropy minus energy, can be defined as an affine upper semi-continuous function over the G-invariant hard core states where G is an invariance group containing space translations. An abstract definition of the pressure and equilibrium states is given in terms of the maximum of the conditional entropy and it is shown that the pressure P_S obtained in this way satisfies $P \geqq P_S \geqq P_\infty$ where P and P_∞ are the thermodynamic pressures obtained from the usual Gibbs formalism with elastic wall, and repulsive wall, boundary conditions respectively. A number of additional results concerning the equilibrium states are also given.

1. Introduction

This paper is a continuation of [11] which we will refer to as I. The purpose of these papers is to attempt to extend results obtained in [1–7][1] to the more general setting of quantum hard core systems. In this second paper we consider the properties of the equilibrium states and show that most of the results of [1–7] can indeed be generalized. The one feature we have not been able to establish is true if the two pressures P and P_∞ introduced in I are equal; thus in effect we reduce the whole problem to this one point, the equality of P and P_∞.

2. Observables and Hard Core States

We will consider particles satisfying Bose-Einstein statistics and leave the easier discussion of Fermi particles to the reader.

[1] We number the references of this paper consecutively with those of I, i.e. Refs. [1–10] should be sought in I.