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Some Physical Region Mass Shell Properties of Renormalized Feynman Integrals

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Abstract. Certain properties proved in S-matrix theory on the basis of macroscopic causality are verified for renormalized Feynman integrals. The technique uses the analytic renormalization of Speer and a specific distortion of the contour of integration. It is proved for an arbitrary Feynman graph G that the corresponding renormalized Feynman integral is holomorphic in the (mass shell) physical region except on the positive-α Landau surface $\mathfrak{L}^+\{G\}$. Under a certain assumption about the geometry of $\mathfrak{L}^+\{G\}$ an "$i\varepsilon$-prescription" is constructed for continuing (in the mass shell) around $\mathfrak{L}^+\{G\}$. The difficulties involved in removing this assumption are discussed.

I. Introduction

Feynman integrals have been an important source of understanding of analytic properties of scattering functions in S-matrix theories [1, 2]. Unfortunately, previous study of the mass shell properties of these integrals has lacked generality because no method was known that dealt efficiently enough with the problems of renormalization. This deficiency can now be repaired with the recently developed techniques of analytic renormalization [3–6].

The object of this work is the verification for renormalized integrals of mass shell properties analogous to those derived for S-matrix elements on the basis of macroscopic causality [7, 8]. The first of these S-matrix properties is that the scattering functions are holomorphic at points of the physical region that are not on some positive-α Landau surface. The second property concerns points on such surfaces. At these points a scattering function can be written as a sum of terms, each of which is the boundary value of a holomorphic function. Usually, only one boundary value term is necessary, but there do exist exceptional points at which two or more are needed [9]. In such situations there is a question of how the singularities are distributed among the various boundary value terms. This is essentially resolved by assumption, the assumption being called the "independence property" [10]. The derivation of this property in the context of Feynman integrals is a second, but presently unrealized, goal of this work.