Commun. math. Phys. 19, 142—160 (1970)

States and Automorphism Groups of Operator Algebras

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Received April 15, 1970

Abstract. Suppose that a group of automorphisms of a von Neumann algebra M, fixes the center elementwise. We show that if this group commutes with the modular (KMS) automorphism group associated with a normal faithful state on M, then this state is left invariant by the group of automorphisms. As a result we obtain a "noncommutative" ergodic theorem. The discrete spectrum of an abelian unitary group acting as automorphisms of M is completely characterized by elements in M. We discuss the KMS condition on the CAR algebra with respect to quasi-free automorphisms and gauge invariant generalized free states. We also obtain a necessary and sufficient condition for the CAR algebra and a quasi-free automorphism group to be η-abelian.

Introduction

Let A be a C^*-algebra, σ_t a one-parameter automorphism group of A, and φ a σ_t-invariant state on A. The state φ is said to satisfy the Kubo-Martin-Schwinger (KMS) boundary condition for $\beta > 0$ if to each $x, y \in A$, there corresponds a function $F(z)$ holomorphic in the strip: $0 < \operatorname{Im} z < \beta$ and bounded on $0 \leqq \operatorname{Im} z \leqq \beta$ with boundary values

$$F(t) = \varphi(\sigma_t(x)\, y) \quad \text{and} \quad F(t + i\beta) = \varphi(y\, \sigma_t(x)) .$$

This condition was first introduced into the "algebraic approach" in [5]. Since that time a great deal of work has been done on the boundary condition. We refer the reader to references [1, 6, 7, 9, 21].

The KMS condition seems to say a good deal about the structure of the algebra involved. For instance it was shown in [17] using Tomita's theory [20] that to every faithful normal state ψ on a von Neumann algebra M there exists a unique one-parameter automorphism group (the modular automorphism group) σ_t^{ψ} satisfying the KMS condition for $\beta = 1$. The structure link here is that M is semi-finite if and only if σ_t^{ψ} is inner [17]. More recently in [18] it was shown that if one has a β-KMS state, ψ, and a γ-KMS state, φ, on a C^*-algebra A, $\beta \neq \gamma$, then the corresponding representations π_{φ} and π_{ψ} are disjoint, provided one representation is of type III.

* The research of R. Herman was supported in part by NSF Grant GP-11475.