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On a Class of Equilibrium States under the Kubo-Martin-Schwinger Condition

II. Bosons

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Abstract. We study equilibrium states of a quantum Bose gas using Kubo-Martin-Schwinger boundary conditions, for a special class of time evolutions, namely the quasi-free evolutions. Under suitable restrictions, in particular positivity of the elementary excitation spectrum, we are able to describe the states fulfilling the Kubo-Martin-Schwinger conditions. In contrast to the Fermi case the solution is, in general, not unique; this is related to a possible Bose condensation.

§ 1. Introduction

In a previous paper [1], we looked for the solutions of the Kubo-Martin-Schwinger boundary condition for Fermi systems and for a special class of evolutions. It is our goal to do a similar study for the Bose systems.

We shall not discuss the importance of the K.M.S. boundary condition within the framework of the algebraic description of equilibrium states of statistical mechanics but only refer to the fundamental papers [2–4], where one can find the formulation of the K.M.S. boundary conditions we shall use.

Definition 1.1. *Let $\mathscr{A}$ be a C*-algebra, $t \to \alpha_t$, a homomorphism of the additive group of reals into *-automorphisms of $\mathscr{A}$: a state ω of $\mathscr{A}$ is said to be a K.M.S. state with respect to $t \to \alpha_t$, if, $\forall A, B \in \mathscr{A}$*

$$t \in R \to \omega(A\,\alpha_t B)$$

can be extended to an analytic function in the strip $0 < \mathrm{Im}\, t < \beta$, continuous on the boundary and such that

$$\omega(A\,\alpha_z B)\big|_{z=i\beta} = \omega(B A)\,.$$

We notice incidentally that a K.M.S. state is automatically an invariant state [3, 5].

Equilibrium states of Bose systems have been extensively studied, especially in the fundamental papers of Araki [6] and Robinson [7];

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