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Estimates of the Unitarity Integral

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Abstract. The elastic unitarity integral is studied for amplitudes which satisfy a Mandelstam representation without subtraction. The double spectral functions are taken to belong to function spaces which allow local, even non-integrable, singularities. The existence of fixed point solutions is derived and the additional restrictions due to inelastic unitarity are discussed.

1. Introduction

The analyticity domain of a two-particle scattering amplitude proposed twelve years ago by Mandelstam [1] has not yet been derived from axiomatic field theory. Also an S matrix theory does not exist which could tell us the analyticity domain of the amplitudes. But a less ambitious consistency problem has been solved; to prove the existence of amplitudes which satisfy Mandelstam analyticity, crossing symmetry for $\pi\,\pi$ scattering, elastic unitarity and the inelastic unitarity bounds [2–6].

The elastic unitarity integral was already written as a system of integral equations for the spectral functions in Mandelstam's original paper [1]. In the case of an unsubtracted amplitude, Atkinson modified these equations to a mapping within a Banach space of Hölder continuous double spectral functions and he showed that its restriction to a subset of this space was a contraction mapping [2]. Hence a fixed point solution which satisfies crossing symmetry and elastic unitarity can be obtained by iteration (under slightly weaker conditions the mere existence of a fixed point can be proved by the Leray-Schauder principle).

This unitarity mapping is not uniquely defined. It depends on an inhomogeneous term which contributes to the double spectral function in the inelastic region $s \geqq 16$, $t \geqq 16$ and which can be chosen arbitrarily within some norm restriction. There is a one-to-one correspondence between this term and the fixed point solution. This arbitrary function might perhaps be determined if one includes all the many particle channels in the unitarity equation, but this is scarcely a solvable problem. However, without too much difficulty one can maintain the unitarity bounds in the inelastic region [as $\operatorname{Im} f_l(s) \geqq |f_l(s)|^2$ in the case of identical particles]. This condition restricts then the choice of the inhomogeneous term.