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The Conservation of Matter in General Relativity

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Abstract. It is shown that in classical general relativity, if space-time is nonempty at one time, it will be nonempty at all times provided that the energy momentum tensor of the matter satisfies a physically reasonable condition. The apparent contradiction with the quantum predictions for the creation and annihilation of matter particles by gravitons is discussed and is shown to arise from the lack of a good energy momentum operator for the matter in an unquantised curved space-time metric.

1. Introduction

In the general theory of relativity it is well known that the equations $T^{ab}{}_{;b} = 0$, which express the local conservation of energy and momentum, cannot be integrated to give conservation laws over a region. This is because the tensor T^{ab} represents the energy-momentum only of matter fields and not of the gravitational field. It is a matter of common experience that the energy of a system is not conserved unless one also takes into account its gravitational energy. In Newtonian theory the concept of gravitational energy is well defined but in general relativity this is unfortunately not the case for arbitrary fields. However, for a bounded system in an asymptotically flat space-time one can define a total energy or mass which represents the energy of both the matter and the gravitational field and which decreases at a rate which can be interpreted as the rate at which energy is carried away to infinity by gravitational radiation [1–4]. The question then arises: could the system radiate away all its mass as gravitational waves and leave just empty space? It will be shown that it could not in the classical theory if T^{ab} satisfies a physically reasonable condition. That is to say a space-time which is non-empty at one time must be non-empty at all times and conversely one which is empty at one time must be empty at all times. The proof depends only on the equations $T^{ab}{}_{;b} = 0$ and not on the field equations.

This result would seem to be in contradiction with the non-zero cross-sections which have been calculated for such processes as the annihilation of a pair of particles into gravitons (see, for example [5]). The reason for this discrepancy between classical and quantum theories