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On the Validity of Ward Identities

P. STICHEL

II. Institut für Theoretische Physik der Universität Hamburg
und
Deutsches Elektronen-Synchrotron DESY, Hamburg

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Abstract. Ward identities for matrix elements of covariant two-point time-ordered operators in the presence of an arbitrary number of subtractions are investigated. Neither the existence of naive T-products nor the existence of equal-time commutators between current densities will be assumed. It is shown by means of the Jost-Lehmann-Dyson representation that T^*-products can always be defined such that normal Ward identities with respect to one current are valid. The simultaneous validity of normal Ward identities with respect to two currents requires a relation between equal-time charge-current commutators.

Our results show that the usual realization of current algebra in the form of Ward identities is possible even if subtractions are necessary. Some examples are discussed in detail.

1. Introduction

Ward identities (WI's) are of great importance for the derivation of low energy theorems [1]. In particular they are the basis for the application of current algebra in the form of the hard-pion methods [2]. Therefore, it is of great interest to investigate in detail the validity of such WI's.

In this paper we attack the problem of the validity of normal WI's for matrix elements of time-ordered products in the framework of general quantum field theory.

We exclusively consider the case of T-products of two field operators where at least one will be a current. Then we speak about a normal WI if a relation of the following kind is valid

$$\begin{aligned}\partial_x^\mu \langle \Psi_2 | T(j_\mu(x)\, A_\gamma(y)) | \Psi_1 \rangle &= \langle \Psi_2 | T(\partial^\mu j_\mu(x)\, A_\gamma(y)) | \Psi_1 \rangle \\ &+ \delta^4(x-y) \langle \Psi_2 | [Q_{(j)}(x_0), A_\gamma(y)] | \Psi_1 \rangle\end{aligned} \qquad \text{(I)}$$

with

$$Q_{(j)}(x_0) \equiv \int d^3x\, j_0(x)$$