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# A Model with Persistent Vacuum* **

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**Abstract.** It is shown that there exists a selfadjoint Hamilton operator corresponding to the interaction $H_0^{(a)} + H_0^{(b)} + \int \Phi_b^+(x)\, \Phi_a(x)\, \Phi_b^-(x)\, d^3x$, where $a$ and $b$ denote two types of scalar particles. We discuss the scattering theory of this operator.

## I. Introduction

It is one of the aims of quantum field theory to describe in a mathematically precise fashion the scattering of relativistic quantized particles. This aim has been reached with different degrees of imperfection for several models of varying complexity. In this article we show that the results known for the Lee model remain essentially valid for a more complex model type to be defined in Section II.

The model treated in this paper is not a relativistic model. It shows, however, the problem of an increasing number of particles and the problem of a logarithmically divergent mass renormalization as the momentum cutoff is removed. The kinematics is relativistic and the interaction is translation invariant. There is no vacuum polarization, and in this sense the model is less ambitious than $\lambda : \Phi^4 :$, and it is at this price that we get more detailed results.

We shall see that this model allows essentially the same conclusions about the existence of scattering states as the Lee model which is very well understood [11] but in which the problem of multiple particle creation is not present. The model we shall treat is very similar to the Nelson model for which much is known about the total Hamiltonian and the $n$-point functions [9, 1]. The Nelson model is somewhat simpler than the present model because the mass renormalization is a logarithmically diverging constant, whereas it is an operator in the case of the persistent model. In Section II we state the definition of the model and give a basic estimate. Section III contains the proof for the selfadjointness of the renormalized total Hamiltonian. The difficult part is the summation of the renormalized Born series for the resolvent. In Section IV we

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