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Small Distance Behaviour in Field Theory and Power Counting

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Abstract. For infinitesimal changes of vertex functions under infinitesimal variation of all renormalized parameters, linear combinations are found such that the net infinitesimal changes of all vertex functions are negligible relative to those functions themselves at large momenta in all orders of renormalized perturbation theory. The resulting linear first order partial differential equations for the asymptotic forms of the vertex functions are, in quantum electrodynamics, solved in terms of one universal function of one variable and one function of one variable for each vertex function whereby, in contrast to the renormalization group treatment of this problem, the universal function is obtained from nonasymptotic considerations. A relation to the breaking of scale invariance in renormalizable theories is described.

Introduction

The small distance behaviour of Green's and vertex functions in renormalizable quantum field theories has been extensively studied in a formal way via the renormalization group [1, 2] and, with equivalent results, some other approaches [8, 9]. Here we offer an alternative approach to the same problem, which appears to be rather more direct. It leads to formulas that are exact and become the usual asymptotic ones upon a, in principle controllable, neglect.

We study the effect of inserting one extra mass vertex, or a generalized mass vertex in the sense of Wilson [3], into all Feynman diagrams for all vertex functions. (Such vertices are defined as those for which the sum of the mass dimensions of the composing fields, a scalar and the electromagnetic field having dimension one, a spinor field dimension three half, a derivative dimension one, is less than four, e.g. two for a scalar mass vertex and three for a spinor mass vertex.) By such insertion, the superficial divergence D of the corresponding Feynman integral is reduced[1] (e.g. by two and by one, respectively, for a scalar and a spinor mass vertex). Reduction of the superficial divergence, however, results in decrease of the large-momentum behaviour by the corresponding power of an overall scale factor[1]. This relation between dimension of

[1] See, e.g., Ref. [2] p. 321; the index $\omega(G)$ we call D. Also ibid., p. 341.