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On Parastatistics

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Abstract. The physical content of a para-Fermi field theory is analysed from the point of view of its local observables. The parafield theory leads to parastatistics only for special choices of the observable algebra, and only then does it give a complete description of the relevant physical states. On the other hand there is always a physically equivalent description in terms of a certain number of ordinary Fermi fields from which the observables are selected by a gauge group (in general non-Abelian). Thus one can always achieve a reduction to Fermi statistics by considering a system with different particle types which are distinguished by hidden (unobservable) quantum numbers.

I. Introduction

Do all particles obey either Bose or Fermi statistics? Specific theoretical models for other alternatives (parastatistics) have been proposed by H. S. Green [1] and studied by various authors[1]. In terms of physical observations the essential feature of such models is the following. Let $\Phi_1, \ldots, \Phi_N$ be different states such that Φ_i is well localized at time $t = 0$ within a space region $\mathscr{V}_i$, the different $\mathscr{V}_i$ being far apart. In the case of ordinary statistics there is exactly one "product state" which corresponds to specifying that observations at $t = 0$ in $\mathscr{V}_i$ find the state Φ_i and observations outside of any $\mathscr{V}_i$ find the vacuum state. In the case of parastatistics, on the other hand, there are several states answering to these specifications. The distinction between these states can be found either by means of a very large measuring apparatus at time $t = 0$ (covering several of the regions $\mathscr{V}_i$) or, more realistically, by measurements at a much later time.

A situation of this sort also arises in another context. Consider the idealized theory of strong interactions in which the electromagnetic and weak interactions are strictly neglected. In such a theory the basic fields have ordinary commutation relations but only those quantities which are invariant under the transformations of a certain symmetry group (the isospin group) can be observed[2]. In particular, the distinction between a single neutron and a single proton becomes impossible; yet

[1] See e.g. [2–4]. There are also various discussions of general statistics which are not directly based on Green's models. Compare for instance [5] and the literature quoted there.

[2] This is so if all possible measuring devices are governed by the laws of the theory. In other words, if the theory does not contain any electromagnetic interactions, then we may exclude the possibility of measurements which rely on electromagnetic effects.