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Scattering Formalism for Non-Localizable Fields

O. STEINMANN
SIN, Zürich

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Abstract. We consider the theory of a non-localizable relativistic quantum field. Non-localizability means that the field is not a tempered distribution, but increases strongly for large momenta. Local commutativity can then not be satisfied. Instead we assume the existence of Green's functions with the usual analyticity properties. We show that in such a theory the S-matrix can be defined, and its elements can be expressed in terms of the fields by the usual reduction formulae.

1. Introduction

It is well known that the fields of relativistic quantum field theory [1, 2] are not functions of the space-time variable x but must be considered as distributions, i.e. as linear functionals over a suitable space of test functions. On the exact nature of the acceptable test functions we have little or no physical information. For mathematical convenience it is usually assumed that the fields are tempered distributions. This assumption may, however, be too restrictive. In particular, there exist indications that the so-called non-renormalizable theories do not fit into this frame-work. Jaffe [3] showed that most of the theorems of axiomatic field theory can be derived for a more general class of fields, with a non-tempered behaviour for large momenta. The allowed increase for large p, is, however, still not arbitrarily fast but is restricted by the condition that the field must be defined in x-space on test functions with compact support. Theories of this type are called "strictly localizable". Strict localizability is necessary if one wants the fields to be locally commuting.

The axiom of local commutativity has, however, no direct physical justification since the usually cited connection with causality is tenuous at best. Close inspection shows that the physically relevant form of causality (macrocausality), at least in its more acceptable formulations, depends mainly on the asymptotic behaviour of the Wightman functions in x-space, and practically not at all on their local properties. Translated into p-space this means that the smoothness properties for finite p are important, while the increase for large p is not. This increase is, then, of little significance. It manifests itself only in a restriction of the allowed test functions.