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The Bargmann-Wigner Method in Galilean Relativity*

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Abstract. The equations of motion of a spin one particle as derived from Levy-Leblond's Galilean formulation of the Bargmann-Wigner equations are examined. Although such an approach is possible for the case of free particles, inconsistencies which closely parallel those encountered in the Bargmann-Wigner equations of special relaticity are shown to occur upon the introduction of minimal electromagnetic coupling. If, however, one considers the vector meson within the Lagrangian formalism of totally symmetric multispinors, it is found that the ten components which describe the vector meson in Minkowski space reduce to seven for the Galilean group and that in this formulation no difficulty occurs for minimal electromagnetic coupling.

More generally it is demonstrated that one can replace Levy-Leblond's version of the Bargmann-Wigner equations by an alternative set which leads to the correct number of variables for the vector meson. A final extension consists in the proof that for all values of the spin the (Lagrangian) multispinor formalism implies the Bargmann-Wigner equations. Thus the problem of special relativity of seeking a Lagrangian formulation of the Bargmann-Wigner set is found to have only a somewhat trivial counterpart in the Galilean case.

I. Introduction

A systematic study of the Galilean group in quantum mechanics has recently been carried out in a series of papers by Levy-Leblond [1–3]. While such an investigation might appear to be of little interest in an era in which special relativity has achieved virtually universal acceptance, it has nonetheless served the highly useful function of pointing out that certain of the predictions of quantum mechanics follow merely from Galilean invariance with no reference whatever to the Lorentz group. Of particular interest in this context is the case of the magnetic moment of the electron which has been shown by Levy-Leblond to have the same value in Galilean relativity as in the Dirac theory [3]. A closely related result is the fact that a nonrelativistic particle cannot possess intrinsic electromagnetic properties other than an electric charge and a magnetic dipole. This then at least suggests that a knowledge of the predictive powers of a non-relativistic theory is not to be disdained and can in fact serve as a basis for a more profound understanding of the impact of special relativity on quantum mechanics.

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