Commun. math. Phys. 18, 65—81 (1970)

Renormalization of a Quadratic Interaction in the Hamiltonian Formalism

J. Ginibre

Laboratoire de Physique Théorique, Faculté des Sciences d'Orsay, France*

and

G. Velo

Istituto di Fisica dell'Università di Bologna

Received February 18, 1970

Abstract. The method of the dressing transformation is used to perform a mass renormalization of a neutral scalar free field in the Hamiltonian formalism, for arbitrary space dimension. The resulting situation is analyzed by means of a Bogoliubov transformation, and seen to yield the expected results.

1. Introduction

It is well known that quantum field theory is plagued with divergences which make the construction of mathematically meaningful models a formidable task. These divergences are of two types: ultraviolet (UV) divergences, connected with high momentum behaviour, and divergences connected with the infinite volume (Haag's theorem).

A significant progress in circumventing the first class of divergences has been made recently by Glimm [1,2] using the hamiltonian formalism. He considered, among others, the case of a neutral scalar field Φ with a Φ^4 interaction in three dimensional space time [2], with a space cut-off which eliminates the infinite volume divergences. The remaining UV divergences still make it impossible to define the Hamiltonian of the system in the original Fock space. However, Glimm was able to define a new Hilbert space in which a suitably renormalized version of the Hamiltonian makes sense as a symmetric operator. Unfortunately, the construction of the new Hilbert space and of the renormalized Hamiltonian is fairly complicated; in particular, it is not known whether the latter is semi-bounded and can be extended to a self-adjoint operator. Moreover, in higher dimensional space time or with more singular interactions, higher divergences occur, and it is not clear how to extend the method to such cases. It is therefore of interest to test the method on a simple model, namely the quadratic Hamiltonian [3, 4], for which nevertheless arbitrarily high divergences occur if one takes the dimension

* Laboratoire associé au C.N.R.S.