Commun. math. Phys. 17, 239—260 (1970)

An Operational Approach to Quantum Probability

E. B. DAVIES*
Massachusetts Institute of Technology, Cambridge, U.S.A.

J. T. LEWIS**
Institute for Advanced Study, Princeton, U.S.A.

Received March 24, 1969; in revised form January 10, 1970

Abstract. In order to provide a mathmatical framework for the process of making repeated measurements on continuous observables in a statistical system we make a mathematical definition of an instrument, a concept which generalises that of an observable and that of an operation. It is then possible to develop such notions as joint and conditional probabilities without any of the commutation conditions needed in the approach via observables. One of the crucial notions is that of repeatability which we show is implicitly assumed in most of the axiomatic treatments of quantum mechanics, but whose abandonment leads to a much more flexible approach to measurement theory.

§ 1. Introduction

It is well known that Kolmogorov's measure-theoretic formulation of classical probability theory [1] can be expressed in a way which emphasizes its similarities with von Neumann's Hilbert space formulation of quantum theory (see for example [2]); the 'observables' of probability theory are the random variables and the 'states' are the probability measures. Many attempts have been made to extend the statistical interpretation of quantum theory by trying to construct analogues of more complicated objects of probability theory, such as joint probability distributions and conditional expectations. The difficulty with these approaches is that it invariably turns out that the relevant objects exist only in very special circumstances; joint distributions in the sense of Urbanik [3] and Varadarajan [4] exist if and only if the observables commute; a conditioning in the sense of Umegaki [5] and Nakamura and Turumaru [6] exists if and only if the observable has discrete spectrum.

* At present on leave from the University of Oxford, Research supported by N.S.F. grant GP-7952X and A.F.O.S.R. contract no. F 44620-67-C-0029.

** At present on leave from Brasenose College, Oxford. Research supported by A.F.O.S.R. grant AF-AFOFR-69-1712.