Commun. math. Phys. 17, 233—238 (1970)

The Commutation Property of a Stationary, Axisymmetric System

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Received December 12, 1969

Abstract. It is shown that in studies of space-time systems which are both stationary and axisymmetric, no generality is lost by considering only cases where the stationary and axisymmetric actions (or equivalently the two corresponding Killing vector fields) commute.

1. Introduction

In discussions of the properties of stationary axisymmetric systems (e.g. in the derivation of Papapetrou's Theorem [1, 2]) it is normally assumed at the outset that the stationary and axisymmetric actions (or equivalently the two corresponding Killing vector fields) commute. Since it is perhaps not immediately obvious (particularly when curved space-time is involved) that this is always justifiable, and since stationary axisymmetric models are so widely used (not only in relativistic and non-relativistic astrophysics, but also in many other branches of physics) it seems worthwhile to demonstrate formally that such an assumption never involves any loss of generality. This is the purport of the theorem which is stated and proved here (which applies whenever the space is sufficiently well behaved to have a continuous Riemann tensor).

The relevant basic properties of axisymmetry and stationary symmetry actions are set out in the definitions and propositions of Sections 3 and 4 in terms of the mathematical framework in Section 2. The theorem itself is given in Section 5.

2. Actions on $\mathscr{M}$

Throughout the whole of this work, it is to be understood that $\mathscr{M}$ (the subject of discussion) is a *connected*, C^3 *n-dimensional manifold* on which is defined a C^2 *Riemannian* or *pseudo-Riemannian* metric and on which further structure (e.g. a Maxwell field, or a field representing stellar matter) may also have been specified. (In most of the physical applications one has in mind, n will be 3 or 4.)