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Continuous Tensor Product States which are Translation Invariant but not Quasi-Free

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Abstract. We show how the theory of continuous tensor products can be used to construct, for commutation relations, translation invariant but not quasi-free states as continuous tensor products of states for systems with one degree of freedom.

Introduction

As was shown by R. T. Powers in [6] § 5.3 for the case of anticommutation relations, all translation invariant states which can be constructed as infinite tensor products of states for systems with a finite number of degrees of freedom are quasi-free and consequently not very interesting for physical applications; in this paper we show how the theory of continuous tensor products allows us to construct, in the case of commutation relations, translation invariant but not quasi-free states as continuous tensor products of states for systems with one degree of freedom; we consider only the nonrelativistic case since, unfortunately, we are not able to carry out the same construction in the relativistic case.

§ 1. The Algebras Associated with a Real Symplectic Space

We consider a real symplectic space (E, σ), i.e. a real vector space E with a non-degenerate symplectic form σ; we call *representation of* (E, σ) every mapping U of E into the unitary operators of a complex Hilbert space such that

(i) for each x in E the mapping $\mathbb{R} \ni h \mapsto U(hx)$ is strongly continuous

(ii) $U(x+y) = e^{i\sigma(x,y)} U(x) U(y)$.

With a real symplectic space one can associate several algebras:

1) The von Neumann algebra $\mathscr{A}_{E,\sigma}$ defined in [2], § 1.3; when E is finite dimensional $\mathscr{A}_{E,\sigma}$ is nothing but $\mathscr{L}(H)$ where H is the space