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Analysis of Conformal-Invariant Scattering Amplitudes for the Two-Dimensional World Model

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Abstract. The explicit form of the "conformal"-invariant "phase shift" analysis in the s-channel for a scattering process involving two incoming and two outgoing particles is derived for the two-dimensional world model. The high energy behaviour of the scattering amplitudes is determined completely (up to a constant factor) by the requirement of "conformal" invariance. It is not possible to obtain this high energy limit by neglecting the masses right from the beginning. The main mathematical problem is the determination of the Clebsch-Gordan coefficients for $SU_0(1,1)$ in the momentum basis.

I. Introduction

In a preceding paper [1] we have discussed a method for the calculation of conformal-invariant scattering amplitudes. Contrary to other authors, who have considered conformal-invariant Lagrangian field theories [2] or off-shell amplitudes [3], we deal with on-shell scattering amplitudes. These scattering processes are not restricted to incoming and outgoing mass zero particles, but include particles with mass $0 < m < \infty$. In this note we should like to present a slightly different analysis for the two-dimensional world model. We assume that the scattering amplitude is not only invariant under the "Poincaré" group $SO_0(1,1) \otimes T_2$, but also under the identity component of the spin-covering group of the group $SO_0(2,2)/C_2$, which is the analogue in our model to the conformal group of relativistic space-time[1]. The identity component of the spin-covering group is given by $SU_0(1,1) \times SU_0(1,1)$; and therefore every Clebsch-Gordan coefficient will separate into a product of two coefficients, which depend on $p^+ = E + p$ and $p^- = E - p$, respectively. Only the irreducible unitary representations of the discrete series [4] $\mathscr{D}^{(+)}_{l/2}$, $l = 1, 2, 3 \ldots$ can possibly be used to describe a physical particle. The restriction of these representations with respect to the translational subgroup [5] shows that the spectrum of p^+ for example is

[1] We use "conformal" group always in this sense.