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# The Time Evolution of Quantized Fields with Bounded Quasi-local Interaction Density

R. F. STREATER and I. F. WILDE*

Mathematics Department, Bedford College, London

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**Abstract.** We extend to arbitrary dimension the proof by Guenin that the time-evolution is an automorphism group of the local algebras, if the interaction Hamiltonian is a space-integral of a bounded local density with finite range.

It has been suggested [1, 2] that, in order to avoid the divergences of quantum field theory, the time-evolution might be regarded as an automorphism group of some $C^*$-algebra $\mathfrak{A}$; if there is a non-trivial interaction, these automorphisms will not be implemented by unitary transformations in the "free" representation of $\mathfrak{A}$.

These ideas have been illustrated in a linear model [1], and in two-dimensional relativistic theories with bounded interaction densities [3]. A similar result has been demonstrated for the Heisenberg ferromagnet and certain fermion systems [4, 5, 8, 9]. In the present paper we offer a generalisation of some of the results of [3] and [5].

We work in the algebraic approach to quantum field theory [2]. More precisely, we make the following assumptions:

1. We are given a $B^*$-algebra $\mathfrak{A}$ of observables, and to each bounded open subset $\mathcal{O}$ of $\mathbb{R}^4$, we are given a sub-$B^*$-algebra $\mathfrak{A}(\mathcal{O})$; we assume that the various $\mathfrak{A}(\mathcal{O})$ generate $\mathfrak{A}$.

2. Causality: if $\mathcal{O}_1$ and $\mathcal{O}_2$ are space-like separated, then $\mathfrak{A}(\mathcal{O}_1)$ commutes with $\mathfrak{A}(\mathcal{O}_2)$.

3. Free field dynamics: we are given a continuous homomorphism, $\tau_0$, from $\mathbb{R}^4$ into the automorphism group of $\mathfrak{A}$, such that $\tau_0(a)\mathfrak{A}(\mathcal{O}) = \mathfrak{A}(\mathcal{O}_a)$, where $\mathcal{O}_a = \{x \in \mathbb{R}^4; x - a \in \mathcal{O}\}$. By continuity we mean that $\|\tau_0(a)A - A\| \to 0$, for any $A \in \mathfrak{A}$, as $a \to 0$ in $\mathbb{R}^4$. For example, $\mathfrak{A}(\mathcal{O})$ could be the $C^*$-algebra generated by a free scalar field $\phi$, smeared with test-functions in $\mathcal{D}(\mathcal{O})$, or that generated by even powers of a free Dirac field in $\mathcal{O}$.

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