Commun. math. Phys. 16, 310—328 (1970)

General Formulation of Griffiths' Inequalities

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Received December 10, 1969

Abstract. We present a general framework in which Griffiths inequalities on the correlations of ferromagnetic spin systems appear as natural consequences of general assumptions. We give a method for the construction of a large class of models satisfying the basic assumptions. Special cases include the Ising model with arbitrary spins, and the plane rotator model. The general theory extends in a straightforward way to the non-commutative (quantum) case, but non-commutative examples satisfying all the assumptions are lacking at the moment.

Introduction

Recently, Griffiths [1] obtained remarkable inequalities for the correlation functions of Ising ferromagnets with two-body interactions. These inequalities were subsequently generalized by Kelly and Sherman [2] to systems with interactions involving an arbitrary number of spins, and by Griffiths to systems with arbitrary spins [3]. These inequalities have received several applications of physical interest. They have been used to prove the existence of the infinite volume limit for the correlation functions of Ising ferromagnets [1], to settle the question of the existence of phase transitions in one dimensional systems with moderately long range interactions [4], to obtain upper and lower bounds on critical temperatures [5], and to establish rigorous inequalities on critical point exponents [6]. It is therefore of interest to extend the inequalities to the largest possible class of models.

In this paper we make a first step in this direction by giving a general formulation which seems appropriate for this problem, both for classical and quantum systems. We obtain sufficient conditions for the inequalities to hold, analyze these conditions, and construct examples which satisfy them. These include as special cases the Ising model with arbitrary spins and the plane rotator model.

In Section 1, we develop the general theory for the classical case. In Section 2, we describe a number of models which fit into the general