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Normal Products in the Thirring Model*

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Abstract. The short-distance behavior of products of fields is studied in the Thirring model of a self-interacting, massless, spinor field in two dimensions. One is able to construct normal products which share most of the convenient properties of the free-field Wick products. A modified form of Wilson's expansion formula is verified. The original formulation of Wilson's hypothesis is seen to hold to every finite order in the coupling constant, but fails in the exact solution of the model for sufficiently strong coupling.

I. Introduction

In relativistic quantum field theory, quantities of physical interest (e.g. currents, Lagrangian densities) are often defined formally in terms of products of field operators at a single point. It is thus of considerable importance to obtain greater mathematical respectability for such objects. In particular, one would like to formulate precisely the transition from the product,

$$A^{(1)}(x_1)A^{(2)}(x_2)\dots A^{(n)}(x_n) \tag{1.1}$$

defined as an n-fold operator-valued distribution, to a local, covariant operator-valued distribution in a single variable (normal product),

$$N[A^{(1)}A^{(2)}\dots A^{(n)}](x). \tag{1.2}$$

A systematic framework for studying the singularity structure of products of field operators has been proposed by Wilson [4], who claims that every product of the type (1.1) should have, for sufficiently small $x_i - x$, an expansion of the form

$$A^{(1)}(x_1)A^{(2)}(x_2)\dots A^{(n)}(x_n) = \sum_k h^{(k)}(x_1, x_2, \dots, x_n)\chi^{(k)}(x) + r(x_1, \dots, x_n) \tag{1.3}$$

where the $h^{(k)}(x_1, x_2, \dots, x_n)$ are complex-valued, possibly singular functions, the $\chi^{(k)}(x)$ are local, covariant fields, and $r(x_1, x_2, \dots x_n)$ vanishes for $x_i = x$, all i. The $\chi^{(k)}(x)$ all may be interpreted as normal

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