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Ground States in Classical Lattice Systems with Hard Core

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Abstract. In this paper we prove the existence of translation invariant ground states in an infinite classical lattice system with hard core and give a characterization of their support. Some examples are discussed.

1. Introduction

In the last years a great deal of effort has been spent on the investigation of equilibrium states of infinite systems in statistical mechanics, classical and quantum lattice systems, and continuous systems of particles with hard cores have been considered; their equilibrium states at temperature $T \neq 0$ have been investigated[1]. In a recent paper [2] Ruelle has started the investigation of the zero temperature states, i.e. the ground states, of the same systems. In this paper we shall study the zero temperature case in a classical lattice system with hard core.

2. Definition of a Classical Lattice System with Hard Core

Consider the lattice $\mathbb{Z}^\nu$, ν being a natural number and $\mathbb{Z}$ denoting the set of all integers. Usually the configuration space for a classical lattice system is taken to be $K = \{0, 1\}^{\mathbb{Z}^\nu}$ which is compact, if we equip $\{0, 1\}$ with the discrete topology (Tychonov's theorem). Each $\hat{X} \in K$ may be interpreted as the characteristic function of a unique set $X \subseteqq \mathbb{Z}^\nu$. Conversely each $X \subseteqq \mathbb{Z}^\nu$ defines a unique $\hat{X} \in K$, its characteristic function. Henceforth we will therefore identify the elements of K with the subsets of $\mathbb{Z}^\nu$. If $x \in X$, we will say that the site x is occupied in the configuration X. In a natural way $\mathbb{Z}^\nu$ acts as a transformation group T on K:

$$T(a): X \to T(a)X = X + a, \qquad X \subseteqq \mathbb{Z}^\nu,\ a \in \mathbb{Z}^\nu .$$

[1] For an account of this, see Ruelle [1] and the literature quoted therein.