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Operations and Measurements. II*

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Abstract. Results of a preceding paper on pure operations are generalized. The application to local field theory is discussed in some detail.

1. Operations

In a previous paper [1] we investigated state changes of a quantum system, called operations.

The state space of the system is a Hilbert space $\mathfrak{H}$, and in the Heisenberg picture used here its state is described by a fixed density operator W, as long as no operations are performed.

An operation was assumed to consist of an interaction of the system with an apparatus, and a subsequent measurement of some property Q' of the apparatus. If $\mathfrak{H}'$ is the state space of the apparatus, W' its initial state, and S the unitary "scattering" operator in $\mathfrak{H} \otimes \mathfrak{H}'$ which describes the interaction, the state W of the system is changed into

$$\tilde{W} = \mathrm{Tr}' \boldsymbol{W}, \qquad \boldsymbol{W} = \frac{\hat{\boldsymbol{W}}}{\mathrm{Tr}\, \hat{\boldsymbol{W}}}, \qquad \hat{\boldsymbol{W}} = (1 \otimes Q')\, \boldsymbol{S}(W \otimes W')\, \boldsymbol{S}^*(1 \otimes Q'). \tag{1}$$

This state change may also be described as

$$\tilde{W} = \frac{\hat{W}}{\mathrm{Tr}\, \hat{W}}, \qquad \hat{W} = \sum_{k \in K} \sum_{i=1}^{n} c_i A_{ki} W A_{ki}^*, \tag{2}$$

with the following definitions [1]. Consider the spectral decomposition

$$W' = \sum_{i=1}^{n} c_i P_{\varphi_i'} \tag{3}$$

with a complete orthonormal system $\{\varphi_i', i = 1 \ldots n\}$ in $\mathfrak{H}'$ [1], $c_i \geqq 0$ and $\sum_{i=1}^{n} c_i = 1$. The subset of all i with $c_i \neq 0$ is denoted by I. Furthermore,

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1 Our discussion applies to finite n as well as to $n = \infty$.