Commun. math. Phys. 16, 1—33 (1970)

Free States of the Canonical Anticommutation Relations*

ROBERT T. POWERS AND ERLING STØRMER**
Department of Mathematics, University of Pennsylvania, Philadelphia

Received September 8, 1969

Abstract. Each gauge invariant generalized free state ω_A of the anticommutation relation algebra over a complex Hilbert space K is characterized by an operator A on K. It is shown that the cyclic representations induced by two gauge invariant generalized free states ω_A and ω_B are quasi-equivalent if and only if the operators $A^{\frac{1}{2}} - B^{\frac{1}{2}}$ and $(I-A)^{\frac{1}{2}} - (I-B)^{\frac{1}{2}}$ are of Hilbert-Schmidt class. The combination of this result with results from the theory of isomorphisms of von Neumann algebras yield necessary and sufficient conditions for the unitary equivalence of the cyclic representations induced by gauge invariant generalized free states.

Introduction

In this paper we study gauge invariant generalized free states of the canonical anticommutation relations, and in particular the question of quasi and unitary equivalence of their induced representations. If K is a separable (complex) Hilbert space $\mathfrak{A}(K)$ – the CAR-algebra of K – is the C^*-algebra generated by elements $a(f)$, where $f \to a(f)$ is a linear map of K into $\mathfrak{A}(K)$ satisfying the canonical anticommutation relations. The gauge invariant generalized free states of $\mathfrak{A}(K)$ are states ω_A whose n-point functions have the structure

$$\omega_A(a(f_n)^* \dots a(f_1)^* \dots a(g_1) \dots a(g_m)) = \delta_{nm} \det((f_i, A g_j))$$

for all $f, g \in K$, where A is a linear operator on K such that $0 \leqq A \leqq I$. These states were first defined and studied by Shale and Stinespring [26].

Since the introduction by Shale and Stinespring [26] generalized free states, which are also called quasi-free states, have been studied by several authors, see e.g. [3, 4, 8, 18, 22, and 24]. Dell'Antonio [8] and Rideau [24] have shown that gauge invariant generalized free states are factor states and have given a characterization of the types of the factors obtained from these states. It follows from their work that the gauge invariant generalized free states ω_A and ω_B are quasi-equivalent if the

* Work supported in part by US Atomic Energy Commission, under Contract AT (30-1)-2171 and by the National Science Foundation.
** Permanent address, University of Oslo, Oslo, Norway.