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On Limits of Separable Potentials and Operator Extensions

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Abstract. A family of self-adjoint Hamiltonians with a separable potential leading towards a contact potential (zero range) is analyzed by tools of functional analysis. It is shown that the family of time evolution operators e^{-iHt} converges strongly (for all t) though the family of Hamiltonians does not converge even weakly. In the case of three dimensions a renormalization procedure is discussed and a correspondence between the renormalized coupling constant and the self-adjoint extensions of the free Hamiltonian is established.

Introduction

The object which we are going to analyze is a one parameter family of Hamiltonians $\{H(\mu)\}_{\mu\in[0,1]} = \{T + V(\mu)\}_{\mu\in[0,1]}$ where $V(\mu)$ denotes a separable potential. We want to give a precise mathematical meaning to the statement, that for $\mu\to 0$ $H(\mu)$ "converges" towards a Hamilton with a contact potential. Furthermore the process of renormalization is a very delicate one from the mathematical point of view and should therefore be studied carefully in the simplest possible case. The detailed study is also necessary as it serves as preparation for the treatment of the problem in the framework of second quantization (to be published in a subsequent paper). We enclose our system in a box of finite length L and assume periodic boundary conditions; the spectra of $H(\mu)$ will therefore be purely discrete for all μ. At first we treat the one-dimensional case which already shows some of the relevant features and study then the case of three dimensions. Our units are $\hbar = 2m = 1$; for simplicity we assume $2\pi/L = 1$, our momenta are therefore $k = n$, n integer or $k = (n_1, n_2, n_3)$ n_i integer, $i = 1, 2, 3$ resp. We use the notation $\rightharpoonup$ and $\to$ for weak and strong convergence resp.

1. The One-Dimensional Case

We consider a one-parameter family of separable potentials $\{V(\mu)\}_{\mu\in[0,1]}$ which approach (in an intuitive sense to be specified) a contact potentials as $\mu\to 0$. In momentum space this is expressed by

$$V(\mu)_{nm} = \lambda\sigma_n^*(\mu)\,\sigma_m(\mu) \quad \text{with} \quad \sigma_n(\mu)\to 1\ \forall n \text{ as } \mu\to 0\,.$$

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