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Projective Unitary Antiunitary Representations of Locally Compact Groups

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Abstract. We give here a systematic presentation of the theory of projective representations when antiunitary operators are present. In particular the imprimitivity theorem of Mackey is proved in this situation and all the unitary antiunitary representations of the extended Poincaré group are derived.

§ 1. Introduction

In the mathematical formulation of quantum mechanics proposed by von Neumann (cf. [3, 8, 9]) the set of all propositions concerning a quantum mechanical system is an orthocomplemented lattice. The simplest example of such a lattice is the lattice $\mathscr{L}(\mathscr{H})$ of all closed subspaces of a separable Hilbert space $\mathscr{H}$. The observables in such a system turn out to be self adjoint operators in $\mathscr{H}$. In order to construct the standard observables like energy, linear, angular and spin angular momenta etc., in such a system it is necessary to study the effect of coordinate transformations by a group G of symmetries. In this context the representations of G in the group $\mathscr{A}(\mathscr{H})$ of automorphisms of the lattice $\mathscr{L}(\mathscr{H})$ is of great importance.

By the extension of a theorem of Wigner (cf. [8], Theorem 7.27, page 167) it is known that every automorphism τ of $\mathscr{L}(\mathscr{H})$ is induced by a unitary or antiunitary operator U^τ on $\mathscr{H}$. The operator U^τ is determined uniquely only up to a scalar multiple of modulus unity. Suppose $\mathscr{U}(\mathscr{H})$ is the group of all unitary and antiunitary operators on $\mathscr{H}$ and $\mathscr{I}(\mathscr{H})$ is the normal subgroup consisting of scalar multiples of identity. Then Wigner's theorem can be reformulated as follows: the group $\mathscr{A}(\mathscr{H})$ is isomorphic with the quotient group $\mathscr{U}(\mathscr{H})/\mathscr{I}(\mathscr{H})$. We shall denote this quotient by $PUA(\mathscr{H})$ and call it *the projective unitary antiunitary group of $\mathscr{H}$*. $\mathscr{U}(\mathscr{H})$ with the weak (operator) topology (which is equivalent to the strong topology) is a complete and separable metric group in which $\mathscr{I}(\mathscr{H})$ is a compact subgroup. Thus the quotient topology in $PUA(\mathscr{H})$ can be carried over to $\mathscr{A}(\mathscr{H})$ in order to make it a topological group.