A General Class of Cut-Off Model Field Theories

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Abstract. We show that Heisenberg picture fields and their vacuum expectation values exist for a wide class of cut-off interactions among fermions and bosons.

I. Introduction

The quantum field theories studied in the present paper include cutoff versions of many standard relativistic quantum field theories. They. have some interest of their own as examples of non-trivial dynamics. However, the main point of studying them is to obtain information about the relativistic theories that are their putative limits as the cutoffs are removed. For this purpose, it is desirable to show

1) that the knowledge of a suitable set of matrix elements of the Hamiltonian of the cutoff theory uniquely determines the one parameter group e^{iHt}, $-\infty < t < \infty$, describing the time evolution of the system,

2) that H has a reasonable spectrum,

3) that the Green's functions of the theory are uniquely determined.

The results of the present paper partially satisfy these requirements. It is shown that for the models considered

1') there is a dense set, D_0, of vectors in the Hilbert space of states in which the Hamiltonian is essentially self-adjoint.

2') that H has a purely discrete spectrum with finite multiplicity, bounded below and is such that its eigen functions lie in D_0.

3') that D_0 is invariant under the smeared fields and that for certain values of the coupling constants the ground state is non-degenerate.

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