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Theory of Filters

BOGDAN MIELNIK
Institute of Mathematical Methods of Physics, Warsaw University, Warsaw

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Abstract. We consider the following statistical problem: suppose we have a light beam and a collection of semi-transparent windows which can be placed in the way of the beam. Assume that we are colour blind and we do not possess any colour sensitive detector. The question is, whether by only measurements of the decrease in the beam intensity in various sequences of windows we can recognize which among our windows are light beam filters absorbing photons according to certain definite rules?

To answer this question a definition of physical systems is formulated independent of "quantum logic" and lattice theory, and a new idea of quantization is proposed. An operational definition of filters is given: in the framework of this definition certain non-orthodox classes of filters are admissible with a geometry incompatible to that assumed in orthodox quantum mechanics. This leads to an extension of the existing quantum mechanical structure generalizing the schemes proposed by Ludwig [10] and the present author [13]. In the resulting theory, the quantum world of orthodox quantum mechanics is not the only possible but is a special member of a vast family of "quantum worlds" mathematically admissible. An approximate classification of these worlds is given, and their possible relation to the quantization of non-linear fields is discussed. It turns out to be obvious that the convex set theory has a similar significance for quantum physics as the Riemannian geometry for space-time physics.

1. Introduction

One of aims of axiomatic quantum mechanics is to provide a most general description of quantum theories. This description usually involves ideal objects called "yes-no measuring devices" or "filters" which are considered elements of an abstract set called "quantum logic". In most papers on the foundations of quantum mechanics the class of measuring devices (filters) is assumed to be given a priori. The properties of this class are described by certain traditional axioms derived from mathematical logic and lattice theory (see [2, 4, 14, 15, 22]). As a result, a certain standard structure is obtained, with filters corresponding to projectors in a Hilbert space and with pure ensembles of quanta being represented by points upon a unit sphere.

This scheme, although useful, is somewhat restricted. There exists a remarkable contrast between the variety of physical phenomena and the homogeneity of a unit sphere. It seems strange that all physical