Commun. math. Phys. 14, 205—234 (1969)

Existence of Phase Transitions for Quantum Lattice Systems

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Received July 7, 1969

Abstract. We prove that the following lattice systems:
(1) anisotropic Heisenberg model,
(2) Ising model with transverse magnetic field,
(3) quantum lattice gas with hard cores extending over nearest neighbours,
exhibit phase transitions if the temperature is sufficiently low and the transverse (or kinetic) part of the interaction sufficiently small.

1. Introduction

The existence of a phase transition at sufficiently low temperature has been proved for a variety of Ising models with attractive interactions [1–7] on ν-dimensional cubic lattices ($\nu \geqq 2$). The argument goes back to Peierls [1]. It can be described in the lattice gas language, and rests on the following ingredients:

(1) *Probability estimate.* With each configuration on the lattice, one associates a family of closed polygonal or polyhedral contours. Let G be such a contour, and g the area of its boundary. One first proves that the probability of occurrence of a given G is bounded by

$$P(G) \leqq \exp[-\beta a g] \tag{1.1}$$

where β is the inverse temperature and a some positive constant.

(2) *Entropy estimate.* The number of possible shapes of G for a given g is bounded by $3^{g-\nu}$.

(3) *Density estimate.* For a given g, the volume enclosed in G is bounded by $(g/2\nu)^{\nu/\nu-1}$, corresponding to the worst possible shape, which is a cube.

From these estimates and general arguments, one deduces the existence of at least two equilibrium states corresponding to the same temperature and chemical potential (or magnetic field), with densities

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