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Gibbs States of a One Dimensional Quantum Lattice

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Abstract. A one dimensional infinite quantum spin lattice with a finite range interaction is studied. The Gibbs state in the infinite volume limit is shown to exist as a primary state of a UHF algebra. The expectation value of any local observables in the state as well as the mean free energy depend analytically on the potential, showing no phase transition. The Gibbs state is an extremal KMS state.

§ 1. Introduction

A one dimensional infinite classical spin lattice system has been studied in [12] and shown to be without any phase transition for a large class of interactions. We show an analogous result for the quantum case with any finite range interaction.

We first show that the power series for the time displacement automorphism of the algebra of observables has an infinite radius of convergence for local observables in one dimensional lattice. This enables us to use the Tomonaga-Schwinger-Dyson perturbation type formula and pull out each potential from $e^{-\beta H}$ as a factor. The transfer matrix technique for the classical one dimensional Ising model is then applicable in a fashion analogous to [12] and we obtain a formula for the infinite volume Gibbs state in terms of an eigen state of a certain linear bounded operator acting on observables.

A standard pertubation theory of bounded linear operators on a Banach space enables us to find an analytic continuation of the Gibbs state with respect to the interaction potential and to prove the analyticity of the expectation value of local observables in the Gibbs state as well as the analyticity of the mean free energy.

This technique is applicable also to the classical case, provided that the interaction potential decreases exponentially at large separation.

The Gibbs state is shown to be invariant under time and lattice translation, satisfies the KMS boundary condition and has the exponen-

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