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On the Most General Linear Theory of Gravitation

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Abstract. The most general field theory of gravitation is analyzed both group theoretically as well as physically. The field equations are solved by means of an algebraic method and it is found that any field theory of gravitation contains only one essential parameter which is correlated to the spin 0 content of the field. Further it turns out that any theory of gravitation must contain a nonvanishing spin 0 part, but general relativity is distinguished by the fact that its spin 0 component cannot be radiated.

After discovery of Einstein's general relativity numerous theories of gravitation have been proposed as alternatives to general relativity (see e.g. [1–4]). It can be shown [5] that all these theories are closely related to one another and differ from general relativity by the presence of a scalar component in the radiation field.

The most general quadratic Lagrangian for gravitation which can be built in terms of the field functions ψ_{ik} and their first derivatives assuming $m_{\text{graviton}} = \emptyset$ is easily decomposed into its several spin parts. Using well known techniques from the theory of representations of the Poincaré group [6, 7] the Lagrangian

$$L(x) = L_0(x) + L'(x)\,, \tag{1}$$

$$L_0 = \sum_{\alpha=1}^{4} c_\alpha I_\alpha\,, \qquad L' = f\psi_{ik}T^{ik}\,, \tag{2}$$

$$I_1 = \psi_{ik,l}\psi^{ik,l}; \quad I_2 = \psi_{ik,l}\psi^{il,k}; \quad I_3 = \psi_{,k}\psi^{,k}; \quad I_4 = \psi_{,k}\psi^{kl}{}_{,l} \tag{3}$$

(f coupling constant, T_{ik} energy-momentum tensor of external mass fields, sum convention is adopted, $\psi = \psi^k_k$) can be decomposed into irreducible parts noticing that a symmetric tensor field $\psi_{ik} = \psi_{ki}$ transforms as

$$\left[D\left(\frac{1}{2}, \frac{1}{2}\right) \times D\left(\frac{1}{2}, \frac{1}{2}\right)\right]_{\text{symmetric part}} = D(1,1) + D(0,0)\,, \tag{4}$$

where $D(1/2, 1/2)$ is the vector representation of the Lorentz group. From there one sees that ψ_{ik} contains four spin parts, namely one of spin 2 and spin 1, resp., as well as two spin 0 parts[1]. The Lagrangian

[1] For details see [8] where we also give a technique to calculate projection operators for the irreducible spin parts for fields having arbitrary discrete spin.