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Irreducible Lie Algebra Extensions of the Poincaré Algebra

I. Extensions with Abelian Kernels

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Abstract. We use cohomology of Lie algebras to analyse the abelian extensions of the Poincaré algebra $\mathscr{P}$. We study particularly the irreducible and truly irreducible extensions: some irreducibility criteria are proved and applied to obtain a classification of types of irreducible abelian extensions of $\mathscr{P}$. We give a characterization of the minimal essential extensions in terms of truly irreducible extensions.

Introduction

The investigation of Lie algebra extensions of the Poincaré algebra has a short history. The only contribution to this analysis is essentially a paper by GALINDO [1]. The more difficult problem of group extensions of the Poincaré group had been discussed formerly by MICHEL [2], in connection with the mixing of internal and space-time symmetry groups. The group extension problem is very hard, especially from the topological point of view, even in the case where only Lie group extensions are considered. This immediately brings about the consideration of Lie algebra extensions. In general, one cannot state that the extensions so obtained have corresponding Lie group extensions. With some connectedness requirements this correspondence can be established [3]. The study of Lie algebra extensions shows up the intrinsic, rather than topological difficulties of the problem. Some manifestations of Lie algebras as fundamental structures in physics suggest also the idea of such an analysis, independently of the corresponding group problem.

We recall in Section I how the cohomology theory of CHEVALLEY-EILENBERG [4] provides for the determination of Lie algebra extensions with abelian kernels [5].

In Section II an important theorem of HOCHSCHILD-SERRE [6] is applied to the study of the abelian extensions of the Poincaré algebra $\mathscr{P}$.

The structure of the Lie algebra obtained by extending $\mathscr{P}$ is analysed in Section III. The irreducibility and true irreducibility of the abelian extensions of $\mathscr{P}$ are examined in Section IV. A classification of types of