Commun. math. Phys. 13, 180—193 (1969)

Limits of Spacetimes

ROBERT GEROCH*
Department of Mathematics, Birkbeck College, London, England

Received April 12, 1969

Abstract. The limits of a one-parameter family of spacetimes are defined, and the properties of such limits discussed. The definition is applied to an investigation of the Schwarzschild solution as a limit of the Reissner-Nordström solution as the charge parameter goes to zero. Two new techniques — rigidity of a geometrical structure and Killing transport — are introduced. Several applications of these two subjects, both to limits and to certain other questions in differential geometry, are discussed.

1. Introduction

One frequently hears statements concerning *the* limit of a family of solutions of EINSTEIN'S equations as some free parameter approaches a certain value. There is, however, a serious ambiguity in such statements, for they normally refer to a particular system of coordinates: by changing coordinates, one can usually obtain some quite different spacetime in the limit. The concept of a limit applied to spacetimes is, nontheless, a useful one, and so we are led to formulate some unambiguous definition of this notion. In this paper we shall define the limits of a family of spacetimes and display a simple characterization of these limits.

In Section 2 we give the definition of a limit. The main theorem of that section asserts that a knowledge of the limit "locally" determines, completely and uniquely, a corresponding global limit. As an example, our definition is applied to clarify the way in which the Reissner-Nordström solution reduces to the Schwarzschild solution as the charge parameter approaches zero.

In Section 3 we discuss those properties of spacetimes which are heriditary, i.e., which pass from a given family of spacetimes to their limits.

The two topics treated in the appendices are useful in many contexts in differential geometry other than merely questions involving limits. The appendices can be read independently of the rest of the paper. In Appendix A we define rigidity of a geometrical structure and prove that nonsingular metrics are rigid. That metrics are rigid while, for example,

* Present address: Department of Physics, Syracuse University, Syracuse, New York.