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A Converse to a Theorem by Friedrichs

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Abstract. It is proved that the requirement of implementability of a group of canonical transformations defines a class of irreducible representations of the CAR. As a corollary a converse to Friedrichs' theorem about canonical transformations implementable in the Fock representation is obtained.

A well known theorem due to K. O. FRIEDRICHS [1] states that a (linear) canonical transformation

$$b^+(f) = a^+(Af) + a^-(Bf) \tag{*}$$

is unitarily implementable in the Fock representation of the canonical anticommutation relations if, and only if, B is of the Hilbert-Schmidt type (i.e.: $B^* B$ is of the trace class).

In this note we prove the following converse theorem: if in an irreducible representation of the canonical anticommutation relations all canonical transformations (*) with $B = 0$ are implementable then it is the Fock or the anti-Fock representation.

Before going further, let us recall the definitions.

Let H be separable Hilbert space (the space of the test functions). By a representation of the canonical anticommutation relations (CAR) over H in a Hilbert space $\mathscr{H}$ we mean a linear mapping $a^+ \colon H \to \mathscr{L}(\mathscr{H})$ such that if $a^-(f) := a^+(f)^*$, $f \in H$, then:

$$a^-(f)\, a^+(g) + a^+(g)\, a^-(f) = (f|g) \quad \text{and} \quad a^+(f)\, a^+(g) + a^+(g)\, a^+(f) = 0 .$$

A Fock (resp.: an anti-Fock) representation of the CAR is an irreducible representation for which there exists $\Omega \in \mathscr{H}$ such, that $a^-(f)\,\Omega = 0$ (resp.: $a^+(f)\,\Omega = 0$) for all $f \in H$.

One says that a pair (A, B), A linear and B an antilinear operators in H, defines a canonical transformation if b^+ defined by (*) is a representation of the CAR.

If, in addition, there exists such unitary $U \in \mathscr{L}(\mathscr{H})$ that

$$b^+(f) = U a^+(f)\, U^{-1}, \quad f \in H .$$

then it is said that canonical transformation is (unitarily) implementable in the given representation.