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The Gravitational Field of Light

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Abstract. I obtain an exact solution of Einstein's equations representing the gravitational field of a steady beam of light. Another exact solution representing two parallel beams shining in the same sense is also given; they do not interact. From a study of null geodesics I conclude that a uniform beam of light is gravitationally stable.

The exact solutions are plane-fronted gravitational waves. It seems that a large class of these waves have as their sources pulses and beams of light.

§ 1. Introduction

Tolman (1934) made an extensive study of the gravitational field of light pulses and beams in the linear approximation to relativity theory. In this paper I give exact solutions corresponding to some of Tolman's approximate ones. The exact solutions confirm one of Tolman's results, namely that a test ray projected parallel to a steady light beam (and moving in the same sense) is undeflected. Indeed I am able to strengthen this to the statement that two steady parallel light beams (shining in the same sense) do not interact, and hence remain parallel.

Tolman also discussed the stability of a light beam, and I take up this problem by studying the null geodesics within the beam, and conclude that a uniform steady beam is gravitationally stable.

An interesting by-product of the work is that the required exact solutions turn out to be metrics of plane-fronted gravitational waves (Brinkmann, 1923; Robinson, 1956; Hély, 1959; Peres, 1959; Takeno, 1961; Kundt, 1961; Ehlers, and Kundt, 1962; Edelen, 1966). Thus we have a physical explanation for a large class of these waves — they are caused by pulses and beams of light.

In § 2 plane-fronted waves and their energy tensor are described, and in § 3 I show how one can construct globally regular solutions representing these waves and their sources. In § 4 I specialise to the stationary case and present a detailed comparison with the work of Tolman: an extraordinary fact becomes apparent here — that the exact and linearised solutions are *identical*. In § 5 I give the exterior and interior field of a uniform steady beam of light, and follow it in § 6 by the field of two such beams (they do not interact). In § 7 two examples of time-dependent fields are given. § 8 contains a detailed investigation of geodesics in the