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Newman-Penrose Approach to Twisting Degenerate Metrics*

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Abstract. The well known method of Newman and Penrose is used to find solutions of the Einstein empty space field equations, which are algebraically special and where the degenerate principal null vectors are not hypersurface orthogonal. As is to be expected the method systematically yields the results obtained by Kerr. An explanation is given of the complex coordinate transformation technique of generating new metrics from Schwarzschild's; also a generalisation of Kerr and Schild type metrics is investigated.

1. Introduction

This paper shows how to find solutions of Einstein's field equations in empty space which contain shear-free, diverging, and twisting geodesic rays.

Such solutions were first dealt with in a general manner by Kerr [1], although he only summarised his results. As is well known, he found an explicit solution which is a rotating generalisation of the Schwarzschild solution. Another explicit solution was found by Newman, Unti, and Tamburino [2], also a generalisation of the Schwarzschild case, though apparantly of less physical interest. Kerr and Schild [3, 4] found a whole class of solutions, including the above Kerr rotating metric, all having a metric tensor of the form:

$$g_{\mu\nu} = \eta_{\mu\nu} + 2\,H l_\mu l_\nu,$$

where $\eta_{\mu\nu}$ is the metric tensor of flat space and l_μ is a null vector. Recently Robinson, Robinson, and Zund [5] have given more details of Kerr's approach, and, as well as making several important simplifications, have found a large class of explicit solutions.

In the following we will repeat the above work using the Newman and Penrose approach [6]. Just as Newman and Tamburino [7] showed how the Robinson-Trautman solutions [8] (the twist-free case) could be found in a straightforward way by this method, we can do the same for the above class — again with not too much difficulty.

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