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Poincaré Partially Integrable Local Representations and Mass-Spectrum

MOSHÉ FLATO and DANIEL STERNHEIMER
Physique Mathématique, Collège de France, Paris — France

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Abstract. An example is given of an irreducible representation of a finite-dimensional Lie algebra containing the Poincaré Lie algebra and giving rise to isolated positive masses. In addition the representation is Poincaré partially integrable (which assures the continuous physical spectrum for the energy- momentum vector) and "Poincaré-covariant" in a weak sense.

A connection between this example and some recently published impossibility theorems is shown, and conclusions about a possible future work in this domain are also drawn.

I. Introductory Remarks and Definitions

Some time ago an example was given [1] of an irreducible representation of a finite-dimensional Lie algebra containing a Poincaré subalgebra for which the squared mass operator is self-adjoint and admits only isolated masses. This example, valid as a counterexample to a no-mass-splitting theorem conjectured earlier [2], suffered from obvious difficulties as to its physical interpretation. In particular the example of [1], which is partially integrable on its translation subalgebra [contrarily of the affirmation of [3]: it is just that the domain of integrability of the translation subalgebra does not coincide with the invariant domain of the entire $\mathfrak{su}(2,2)$ Lie algebra; in that example the energy-momentum P_μ is taken on its maximal domain of definition], is not integrable on the Lorentz part of $\mathfrak{su}(2,2)$. Would it be Poincaré-partially-integrable [4] then according to [5] the spectrum of the energy-momentum vector P_μ would have to be continuous. The fact that the irreducible representations of the translation group are all one-dimensional shows that there is no surprise in the fact that in [1] the spectrum of P_μ admits gaps of the order of magnitude of the mass-splitting (a fact which was also mentioned by [3]).

In [3] a corollary of a rigorous no-go theorem (applied to finite dimensional *Lie groups* and proved by JOST and SEGAL [6]) is proved which makes it possible to apply the result of [6] to a very restricted family of non-integrable (or local) representations of Lie algebras. It is only the fact that one could think (as it is claimed in [3]) that practically