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# On the Mathematical Structure of Einstein's Equations in Mixed Initial and Boundary Value Problems

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**Abstract.** It is shown that, also in the mixed initial and boundary value problem, Einstein's equations may be replaced by the two subsystems $T_l^m//_m = 0$ and $R_{\alpha\beta} = -\varkappa\left(T_{\alpha\beta} - \frac{1}{2} T g_{\alpha\beta}\right)$, provided that the initial data verify the consistency conditions $G_i^4 = -\varkappa T_i^4$ and that the analogous relations

$$(G_{lm} + \varkappa T_{lm}) N^m = 0$$

are imposed on the boundaries of the given domain.

## Introduction

Einstein's gravitational equations[1]

$$G_{lm} = -\varkappa T_{lm} \tag{1}$$

imply the four relations

$$T_l^m //_m = 0 . \tag{2}$$

Conversely, LICHNEROWICZ [1] (see also [2, 3]) has proved that, in the initial value problem, Eq. (1) may be replaced by (2) and by the system

$$R_{\alpha\beta} = -\varkappa_{\alpha\beta}\left(T_{\alpha\beta} - \frac{1}{2} T g_{\alpha\beta}\right) \tag{3}$$

provided that the initial data verify the consistency conditions

$$G_i^4 = -\varkappa T_i^4 . \tag{4}$$

The importance of this result lies in the fact that it singles out Eq. (2) as a subsystem of Eq. (1). Now, since Eq. (2) have a definite physical meaning by themselves, the above result allows us to study their role in the determination of the solution of the gravitational problem.

For the same reason, an analogous possibility is also desirable in more general cases, e.g. in the mixed initial and boundary value problem. In fact, as we shall prove in a later paper, the analysis of Eq. (2) in a spatially finite domain of space-time (e.g. in a four-dimensional "world-

[1] Latin indices run from 1 to 4. Greek indices run from 1 to 3. The metric tensor is assumed to have the signature +2. A comma indicates partial derivative; a double stroke [like in Eq. (2)] indicates covariant derivative.