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Boson Fields Under a General Class of Cut-Off Interactions

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Abstract. We consider a boson field $\phi(x)$ under an interaction of the form $\int\limits_{|x|\leqq r} V(\phi_k(x))\,dx$, where $\phi_k(x)$ is the momentum cut-off field, and $V(\alpha)$ is a continuous bounded function. Under a weak regularity condition on $V(\alpha)$, we prove that the total energy operator is self adjoint, that the asymptotic fields exist and that the scattering operator exists.

1. Introduction

The object of this paper is to study a wide class of quantum fields where the interaction is given by a local relativistic interaction with a momentum and space cut-off. The fields will be self interacting boson fields, where the energy operator is given in the form

$$H = H_0 + \int\limits_{|x|\leqq r} V(\phi_k(x))\,dx\,.$$

H_0 is the free energy operator of a free boson field of strictly positive mass m. $V(\alpha)$ is a real function of a real variable α, such that $V(\alpha)$ is the Fourier transform of a finite measure. $\phi_k(x)$ is the free field with a momentum cut-off at k. In two dimensional space time GLIMM [2] has investigated the case where V is a semibounded polynomial, and he was in this case able to remove the momentum cut-off. The case where $V(\alpha) = \lambda\alpha^4$ and still in dimension two, can be treated more thoroughly, as shown by JAFFE and GLIMM [3].

One can in this case after removal of the momentum cut-off prove that the total energy operator is self adjoint on the intersection of the domains of its free and interacting part. For the case where $V(\alpha)$ is a semibounded polynomial but in dimension four and with a momentum cut-off JAFFE, LANFORD and WIGHTMAN [4] were able to prove that the total energy is a self adjoint operator.

We shall prove that in the case V is the Fourier transform of a finite measure, the interaction will be bounded, and so there is no problem with the self adjointness of H. But the main object of this paper is to prove existence of asymptotic fields, and the existence of the scattering