Commun. math. Phys. 12, 212—215 (1969)

Non-Existence of Spontaneous Magnetization in a One-Dimensional Ising Ferromagnet

FREEMAN J. DYSON
Institute for Advanced Study, Princeton, New Jersey

Received January 13, 1969

Abstract. It is proved that an infinite linear chain of spins $\mu_j = \pm 1$, with an interaction energy

$$H = -\sum J(i-j)\,\mu_i\mu_j\,,$$

has zero spontaneous magnetization at all finite temperatures, provided that $J(n)$ is non-negative and that

$$(\log\log N)^{-1}\sum_1^N n\,J(n) \to 0 \quad \text{as} \quad N \to \infty\,.$$

This shows that a theorem of RUELLE, establishing the absence of long-range order when the sum $\sum n\,J(n)$ converges, is not the best possible.

1. Result

This paper is a sequel to an earlier one [1] dealing with the existence of phase-transitions in the infinite Ising ferromagnet with energy

$$H = -\sum_{i>j} J(i-j)\,\mu_i\,\mu_j\,. \tag{1.1}$$

In [1] it was proved that a transition at a finite temperature from zero to nonzero spontaneous magnetization does occur if $J(n)$ is positive and monotonically decreasing and if

$$M_0 = \sum_{n=1}^{\infty} J(n) < \infty\,, \tag{1.2}$$

$$K_3' = \sum_{n=1}^{\infty} (\log\log(n+4))\,[n^3 J(n)]^{-1} < \infty\,. \tag{1.3}$$

On the other hand, RUELLE [2] has proved that if $J(n)$ is positive and

$$M_1 = \sum_{n=1}^{\infty} n\,J(n) < \infty\,, \tag{1.4}$$

then there is zero spontaneous magnetization at all temperatures. A gap remains between the conditions (1.3) and (1.4), including the particularly interesting case

$$J(n) = n^{-2}\,. \tag{1.5}$$