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Scattering into Cones I: Potential Scattering

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Abstract. In the non-relativistic time-dependent theory of scattering, we compute the probability that a particle with given initial state should scatter into a cone with apex at the origin. A formula for this probability is found which holds good for a large class of short-range potentials and also for Coulomb potentials. The formula is obtained by showing that if the initial state of a particle is specified by f, then its position probability density at large positive times can be taken to be $|(e^{-2iHt} e^{iH_0 t} f)(\boldsymbol{x})|^2$.

Introduction

In the time-independent theory of non-relativistic potential scattering, the principal object of interest is the differential scattering cross-section $\sigma(\theta, \emptyset)$. This quantity has a very simple interpretation, as it is the probability density that an incident particle will be deflected through a certain angle during the scattering process. Typically, in the time-dependent theory, one loses sight of the simple geometrical features of the scattering process and concentrates attention on the computation of transition probabilities from one state to another. In this paper, we will emphasize geometrical features in the time-dependent theory by computing the probability that a particle with given initial wave-function will be scattered into a cone C with apex at the origin of coordinates. This quantity has an easily visualizable meaning, and it seems a natural thing to calculate in view of typical experimental setups. The formula obtained for this probability can be put in a simple form which is applicable both to scattering problems involving a large class of short-range potentials and to scattering by Coulomb potentials. The formula also admits of a generalization to problems involving n particles. In this paper we discuss the case of potential scattering. n-body problems will be dealt with in a forthcoming paper.

Potential Scattering into Cones

Orientation. We describe a nonrelativistic particle by a wave-function, i.e. a normalized element Ψ of $\mathscr{L}^2(\mathbb{R}^3)$, the Hilbert space of complex valued square-integrable functions on three-dimensional Euclidean

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