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Quasi-Unitary Algebras Attached to Temperature States in Statistical Mechanics. A Comment on the Work of Haag, Hugenholtz and Winnink

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Abstract. We show that the *-algebra of "analytic elements" with respect to time translations which plays a central role in HAAG, HUGENHOLTZ and WINNINK'S formulation of the Kubo-Martin-Schwinger boundary condition, is a quasi-unitary algebra in the sense of DIXMIER. The commutant theorem proved by HAAG, HUGENHOLTZ and WINNINK is thus reduced to DIXMIER'S commutant theorem for quasi-unitary algebras.

1. Introduction

In a very interesting paper [1] (referred to below as HHW), HAAG, HUGENHOLTZ and WINNINK describe general features of the equilibrium states of quantum statistical mechanics at finite temperature. A state is viewed as normalized positive linear functional ω on a C^*-algebra $\mathfrak{A}$ of quasi-local observables. Time evolution is described by a one-parameter group, $t \to \alpha_t$, of automorphisms of $\mathfrak{A}$. An algebraic formulation of the Kubo-Martin-Schwinger [2, 3] boundary condition is given as a property of equilibrium states with respect to the time-development automorphisms. Furthermore, it is shown that, in contrast to the zero temperature situation, the representation of $\mathfrak{A}$ obtained from an equilibrium state ω by means of the Gelfand-Segal construction is reducible, the corresponding weak closure being one-to-one with its commutant.

The main mathematical tool in HHW is a norm-dense *-subalgebra $\tilde{\mathfrak{A}}$ of "analytic elements" of $\mathfrak{A}$. The purpose of the present note is twofold. First, we fix some points of rigor in HHW using the necessary amount of vectorial distributions: to each C^*-algebra $\mathfrak{A}$ with an abelian