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Existence of a Phase-Transition in a One-Dimensional Ising Ferromagnet

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Abstract. Existence of a phase-transition is proved for an infinite linear chain of spins $\mu_j = \pm 1$, with an interaction energy

$$H = - \sum J(i-j)\,\mu_i \mu_j\,,$$

where $J(n)$ is positive and monotone decreasing, and the sums $\sum J(n)$ and $\sum (\log\log n)\,[n^3 J(n)]^{-1}$ both converge. In particular, as conjectured by KAC and THOMPSON, a transition exists for $J(n) = n^{-\alpha}$ when $1 < \alpha < 2$. A possible extension of these results to Heisenberg ferromagnets is discussed.

I. Introduction

We consider the one-dimensional Ising ferromagnet with sites labeled by an integer j taking all values from $-\infty$ to $+\infty$. At each site is a random variable μ_j taking the values ± 1, the total energy being

$$H = - \sum_{i>j} J(i-j)\,\mu_i \mu_j\,, \tag{1.1}$$

with

$$J(n) \geqq 0\,, \quad n = 1, 2, 3, \ldots \tag{1.2}$$

GALLAVOTTI and MIRACLE-SOLE [1] have proved that this system exists as a well-defined limit of a finite system, allowing a consistent definition of thermodynamic averages, provided that

$$M_0 = \sum_{n=1}^{\infty} J(n) \tag{1.3}$$

is finite. Since we are assuming (1.2), the case in which M_0 is infinite is mathematically uninteresting. When M_0 is infinite there is an infinite energy-gap between the ground states and all other states, so that the system is completely ordered at all finite temperatures, and there can be no question of a phase-transition.

On the other hand, the case in which only a finite number of $J(n)$ are nonzero has long been known [2] to be uninteresting for the opposite reason; the system can have no phase-transition because it is disordered at all finite temperatures. An interesting one-dimensional model considered by BAUR and NOSANOW [3], giving rise to a phase-transition