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On the Product Form of Quasi-Free States

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Abstract. The product form of quasi-free states is outlined, and the types of the generated factors are exhibited whenever the states are translation invariant. Among these states some are shown to be involved in the study of Fermi and Bose gases.

1. Introduction

The "quasi-free states" originated from the "generalized free fields" introduced by O. W. GREENBERG [1]. They were defined and studied in references [2—7].

Whenever the quasi-free states of the C^*-algebra of commutation relations or of anticommutation relations are examined the papers [5] or [6] are referred to.

In Section 2, the fermion case is considered; it is chiefly shown that any quasi-free state is, with the meaning given by POWERS [3], a product state of partial states. These latter ones are primary if the state is translation-invariant and their types are exhibited.

An analogous analysis is made in Section 3 in the boson case, and similar results are obtained.

Finally we conclude by showing the physical significance of some quasi-free states involved in the study of Fermi and Bose gases.

2. Fermions

2.1. Generalities

Let (H, s) be a real Hilbert space of finite or infinite, but countable dimension, equipped with a scalar product:

$$(\psi, \varphi) \in H \times H \to s(\psi, \varphi) \in R$$

(one-particle space). Consider the Clifford algebra $\mathfrak{A}(H, s)$ built on (H, s); that is an involutive algebra with unit element (denoted by 1), and generated by the set of elements $B(\psi)$, linear with respect to ψ, which satisfy the anticommutation relations:

$$[B(\psi), B(\varphi)]_+ = 2s(\psi, \varphi)\, 1 \,.$$

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