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On the Quantum Logic Approach to Quantum Mechanics

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Abstract. A quantum logic structure for quantum mechanics which contains the concepts of a physical space, localizability, and symmetry groups is formulated. It is shown that there is an underlying Hilbert space which mirrors much of this axiomatic structure. Quantum fields are defined and shown to arise naturally from the quantum logic structure. The fields of HAAG and WIGHTMAN are generalized to this theory and an attempt is made to find a local equivalence for these fields.

1. Introduction

The two main methods of attack in axiomatic quantum mechanics have been the c^*-algebra [1] and the quantum logic [2] approaches. The first of these inequivalent approaches uses the algebra of bounded observables as the main axiomatic elements while the basic constituents of the second are the quantum propositions or, as they are also called, experimental questions or events. In this paper we are concerned with the second approach.

One of the main goals of the quantum logic approach is to postulate enough physically verifiable axioms so that the structure of the proposition system reduces to the usual von Neumann Hilbert space model for quantum mechanics. In the author's opinion this goal has not been achieved. In all such attempts axioms, such as completeness and atomicity, have been imposed [3] although these axioms have little physical justification. Even the lattice structure of the proposition system seems questionable [4]. For this reason we shall not impose these questionable axioms and use only postulates which seem physically reasonable and justifiable and which hopefully can be tested in the laboratory. Even so, as we shall see, we are very close to a Hilbert space theory as far as the structure of the axiomatic system is concerned. We will also show that even in this abstract context many of the constructs, such as localizability, symmetry groups, and quantum fields, which are used in general quantum theory may be formulated and in fact arise quite naturally.

2. The Axiomatic Structure

Suppose S is a quantum mechanical system. Let us now analyze S and attempt to extract from it the physically relevant properties it